

Estimating Human Movements Using Memory of Errors

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Abstract

Humans estimate their movements based on their knowledge of the dynamics of the environment, and on actual sensory data. Wolpert and colleagues have incorporated this understanding into a model that simulates this estimation using the Kalman filter [1]. Inspired by a recent study in neuroscience [2], we here introduce a new factor—memory of errors—into this simulation of movement estimation. These historical errors help humans determine the stability of the environment, which could be either steady or rapidly changing. This condition controls the rate at which a given error will be learned, so as to affect the estimates of future movements. We here apply our new model, a modified Kalman filter incorporating memory of errors, to the simulation of a hand lifting movement, and compare the simulated estimation process with its human counterpart.

Keywords: sensorimotor system, estimation of movements, internal model, Kalman filter, memory of errors, LIDA model, cognitive model

1 Introduction

Wolpert and his colleagues [1] have investigated a sensorimotor integration mechanism by which people produce an estimate of the result of their movement. They have hypothesized that the central nervous system (CNS) internally predicts the result of a self-produced movement by simulating the dynamics of the environment* using a so-called (forward) internal model; this prediction is then combined with a reafferent sensory correction [1]. To test this hypothesis, they have simulated this prediction and correction using the Kalman filter [3]. In this way, they qualitatively replicated how humans estimate their hand movements in the dark.

The question of whether combining such an internal model with sensory correction is in fact neurally implemented in humans, or is just a metaphor for what the human nervous system does, remains open [4]. However, this model is useful for studying further hypotheses, including Bayesian

* The environment includes both the agent's motor system (body) and the world the agent lives in.

decision theory for sensorimotor control [5], optimal feedback control [6], and motor recognition [7]. Moreover, the Kalman filter itself has been applied in different domains in other fields. together with its extended version: extended Kalman filter (EKF) [8].

In the Kalman Filter, there are two factors that balance the importance between predicted results and sensory results: the inaccuracy in the knowledge of the dynamics of the environment, and the noise in the sensory process.

Here we introduce a third balancing factor, changes in the environmental dynamics. Actually, humans may experience, and then remember, such changes as a kind of error, the difference between intended (predicted) results and actual (sensory) results. We propose that this new factor is driven by memory of errors caused by changes in the dynamics. This idea is inspired by a recent study in sensorimotor learning [2]. Herzfeld and his colleagues hypothesize that besides learning from errors, the brain may decide how much to learn from a given error depending on its memory of errors. These historical errors help humans determine whether the environment is steady or rapidly changing. Environmental stability thus controls how much of a given error will be learned so as to affect the estimate of the upcoming movement.

In the following section we describe and compare the studies of Wolpert et al. [1] and Herzfeld et al. [2]. We then introduce our new model, a modified Kalman filter, which estimates human movements using memory of errors, and go on to describe a computational experiment that simulating hand lifting action. Finally, we propose some directions for further research.

2 Previous Work

In this section we first review a study regarding how people estimate their hand movements in the dark [1], and then introduce a recent study about how memory of errors affects sensorimotor learning [2]. Finally, we compare the two studies. In this way, we provide adequate background knowledge to prepare for the introduction of our new model in the following section.

2.1 Simulating a Sensorimotor Integration Process Using the Kalman Filter

Wolpert and colleagues [1] have argued for the existence of an internal model in the central nervous system (CNS) that simulates the response of the motor system. They have carried out a human experiment in which participants move one of their hands horizontally on a plane either to the left or to the right along one dimension in the dark. In the absence of vision, their sensory feedback consists only of proprioception during the movement. Participants are instructed to continue moving until they hear a tone. The timing of the tone is controlled so as to produce a uniform distribution in movement duration between 0 and 3 seconds. At the end of each movement (trial), participants indicate (estimate) the unseen new location of their moved hand. The difference (error) between participants' real and estimated new hand locations is recorded as a function of movement duration. In total, eight participants performed 300 trials each. In this experiment, researchers found that on average, 1) participants overestimate their hand locations—the estimated movement duration is longer than the actual duration—and 2) the error peaks after one second and then decreases gradually.

As argued by Wolpert et al. [1], “these experimental results can be fully accounted for if we assume that the motor control system integrates the efferent outflow and the reafferent sensory inflow”. To support this conclusion, they developed a computational internal model, with the use of a reafferent sensory correction, to replicate human self-estimation of hand movements in the dark using a Kalman filter.

The Kalman filter is a recursive algorithm that estimates the state of a discrete-time linear stochastic system [3, 9]. It first predicts the system's next state in the timeline, based on its current

state, on knowledge of the running system's dynamics, and optionally on its current motor command. Then it corrects the prediction based on sensory data that may have noise. This two-step routine operates iteratively online to estimate the system's state. From a mathematical viewpoint, the Kalman Filter is a set of equations that provides an efficient estimate for the state of a process, expressed by Eqs. (1) ~ (5).

$$\dot{x}_t = Ax_{t-1} + Bu_t \quad (1)$$

$$P_t = P_{t-1} + Q \quad (2)$$

$$K_t = P_t / (P_t + R) \quad (3)$$

$$x_t = \dot{x}_t + K_t(Cz_t - \dot{x}_t) \quad (4)$$

$$P_t = (1 - K_t) P_t \quad (5)$$

The Kalman Filter's prediction process is represented by Eqs. (1) and (2), and its correction process is represented by Eqs. (3) ~ (5). Variable x represents the state value. Specifically, x_{t-1} , \dot{x}_t , and x_t represent the immediately previous, intermediate predicted, and current estimated state values respectively. Variable u_t represents the input motor commands, and z_t the input sensory data. A , B , and C are the parameters for the above variables. K acts as a gain that weights the new sensory data against the predicted result. Parameters Q , R , and P represent the uncertainty of the prediction, the correction, and the entire estimation respectively. For further details, [3], [9], or [10] may be consulted. From one viewpoint, the Kalman filter is a kind of non-Markovian extension [11] because its estimation relies on its historical data, while optimality is not of concern, and so is not guaranteed in our new model.

Based on the simulated results, Wolpert et al. [1] have shown that the Kalman filter is able to qualitatively reproduce the propagation of the error of the estimated hand location as a function of movement duration.

2.2 A Memory of Errors

In the study of sensorimotor learning, Herzfeld and colleagues [2] have hypothesized that the brain not only learns from individual errors that it has experienced before, but also accumulates the errors into a memory; this memory of errors makes it possible for the brain to control how much it will learn from a given current error.

Herzfeld et al. [2] have done human experiments to explore the effect of memory of errors in human hand-reaching movements. The experimental setup is as follows. (1) A participant sits down in front of a table, and holds the handle of a robotic arm; the arm is attached on the table, and its handle can be moved because of several moveable joints in the arm. The participant is asked to repeatedly make out-and-back reaching movements; the goal for a trial is to reach a target location from an initial location. (2) The participant's hand is occluded by an opaque horizontal screen that is located above the plane of the forearm; thus the participant cannot see his hand. (3) An overhead projector displays information on the screen about the actual hand location, the initial location, and the intended target location of the reach. This information is visually available to the participants. (4) During a reaching movement (only on the outward reach), the participant's hand may be perturbed by the robotic arm through its handle with a force perpendicular to the reaching direction. The perturbation produces an error during the reaching movement, the difference between the intended hand location and the actual hand location upon arriving. (5) The magnitude of the perturbing force is constant, and the direction may be either to the left or to the right. Thus the force may create two types of errors.

Using this experiment, Herzfeld et al. examine the relationship between memory of errors, and the amount that is learned from a given error. They hypothesize as follows: "[Consider] an environment in which the perturbations persist from trial to trial, and another environment in which the perturbations switch ... In a slowly switching environment, the brain should learn from error because the

perturbations are likely to persist (learning from error in one trial will improve performance on the subsequent trial). However, in a rapidly switching environment, the brain should suppress learning from error because any learning will be detrimental to performance on subsequent trials” [2].

In the experiment, participants are randomly divided by environmental stability into three groups (9 per group): they first performed 30 trials of reaching in either a slowly, medium-speed, or rapidly switching environment—the direction of the perturbing force switches. And then all participants experience a pure reaching movement without any perturbation for 10 trials[†]; in this way, the effects of the perturbation are removed. Finally, all participants experience one reaching trial with the same perturbation.

The researchers measured the change in the force applied by participants before and after the final perturbation. By considering the force produced by a participant a proxy for the participant’s estimate of the perturbing force, they can indirectly measure how much the participant’s estimate of the perturbing force has been updated after experiencing a perturbation. They found that the responses of participants to the same perturbation are different between groups. A participant gives larger responses—corresponding to a higher estimate of the force—in the slowly switching environment and smaller responses—indicating a lower estimate of the force—in the rapidly switching environment. This phenomenon supports their hypothesis quoted above.

Note that in this experiment, although the memory of perturbation has been removed using 10 trials of pure reaching movements before measuring the effect of the final perturbation, a more abstract attribute of the environment, corresponding to a level of persistence of the environment—environmental stability, is still available in the memory, and thus influences the effect of the final perturbation. A term “saving” names the influence of this available abstract attribute.

Because of space limitations, we did not explain every detail of the experiment above. A full explanation can be found in the original paper [2].

2.3 Comparisons between the Two Studies

In this subsection, we compare the two studies reviewed above. To conserve words, we cite the two papers [1, 2] for the two studies respectively in the whole subsection here, and at times below we simply refer to the study of Wolpert et al. [1] as the first study and to the study of Herzfeld et al. [2] as the second study.

First, in both studies, researchers investigate the process by which humans produce an estimate of their movement. Wolpert and colleagues simulate how people estimate their hand move in the dark using the Kalman filter, and Herzfeld and colleagues propose a causal relationship from memory of errors to the knowledge of the environmental dynamics, which knowledge affects the estimation of upcoming movements.

However, the estimation processes examined in the two studies are at different levels. Wolpert and colleagues study the estimated hand location within a single movement trial. They calculated the propagation of estimation error on average for one movement, while they did not concern themselves with the relationships between multiple movement trials. On the other hand, Herzfeld and colleagues study the estimated hand location between trials. They proposed the hypothesis regarding the effect of historical movements on the estimation of the current movement. But we still consider these two studies comparable, because in fact, they are qualitatively studying the same thing, how humans estimate their movements. From this viewpoint, it is reasonable to borrow ideas from the second study to modify the simulation implemented in the first study.

Second, in both studies, an update process relying on an error—the difference between predicted (intended) results and sensory (actual) results—is used in the process of producing the estimate of movements. In the first study, the predicted result is corrected using sensory results. A parameter K is used to weight the effect of the error in this correction (see Eq. (4)). The value of K depends on both

[†] This pure reaching movement is known as a “washout” (Herzfeld, Vaswani et al. 2014).

the inaccuracy of the knowledge of the environmental dynamics, and the noise in the sensory process (see Eq. (3)). In the second study, a memory of errors controls (weights) how much the current error will be used for updating the newly estimated result, the magnitude of a type of learning rate. Here we see that in the first study, there are two factors—the inaccuracy and the noise—that weight the error, and in the second study, a third factor—memory of errors—is used.

3 A Model That Estimates Human Movements Using Memory of Errors

In this section, we first propose an operational definition for a learning rate η that determines how memory of errors functionally controls the extent to which errors will be learned. This definition is conceptually inspired by the work of Herzfeld et al. [2]. We then introduce a modified Kalman filter, in which we add a new factor—memory of errors—to balance the importance between predicted results and sensory results. The effect of this new factor is represented by the magnitude of the learning rate η . In this way, we achieve a new model that is able to reflect its knowledge of memory of errors—a feature of the environmental dynamics—into the process of producing movement estimates.

3.1 The Learning Rate η

The magnitude of the learning rate (η) is controlled by memory of errors. The specific formula for this control is represented as a sigmoid function expressed by Eq. (6), and which is assisted by Eq. (7). The learning rate ranges from 0.5 to 1.5 with a default value of 1.0.

$$\eta = \frac{1}{1 + e^{-\theta t}} + 0.5 \quad (6)$$

$$t = 1.0 - \frac{2s}{n} \text{ if } n \neq 0, \text{ and } t = 0 \text{ if } n = 0 \quad (7)$$

Specifically in Eq. (6), the variable t represents the status of the memory of errors, which is calculated according to Eq. (7). It ranges from -1.0 to 1.0. The parameter θ tunes the effect of t , and is set to 6.0 by default. In Eq. (7), the variable n represents how many errors have been experienced by the brain, and thus stored in the memory. Variable n is an integer starting from zero.

As mentioned in Section 2.2, the forces of perturbations used in the study of Herzfeld et al. [2] have the same magnitude with directions either to the left or to the right. Similarly, we set only two types of errors in our model: the same magnitude with either positive or negative sign. Variable s represents how many times the error type has switched within the memory of errors. Variable s is an integer starting from zero.

Here we explain the behavior of the above formula with examples. If the brain has experienced many errors and most of them have the same sign, the value of n is large and the value of s is small; therefore, the value of t is large, close to 1, so that the value of θt is close to 6.0 and η is close to 1.5. This means a slowly switching environment results in a high learning rate—learning more from the current error. On the other hand, if there are many errors in memory and they have switched signs very often, the values of both s and n are large, so t is negative with a large absolute value; thus the value of η is close to 0.5. This means a rapidly switching environment leads to a low learning rate—learning less from the current error. These simulated behaviors qualitatively agree with the hypothesis proposed by Herzfeld et al. [2]. Note that when there is no error in the memory yet, the value of t is 0 because n is 0, so the value of η is 1.0, which is considered the default value of η .

3.2 A Modified Kalman Filter

Compared to the original Kalman filter expressed by Eqs. (1) ~ (5), we modified Eq. (4) by adding a new variable η , which is defined in Section 3.1 above, as expressed by Eq. (8). The newly modified Kalman filter is expressed by Eqs. (1)~(3), (5), and (8).

$$x_t = \hat{x}_t + \eta K_t (Cz_t - \hat{x}_t) \quad (8)$$

The added variable η represents a new factor that balances the importance between predicted results and sensory results, occurring together with the parameter K .

Two questions need answering regarding this modified Kalman filter: does this modification make sense, and what is its benefit? For the first question, as we have discussed in Section 2.3, both of the studies [1, 2] introduce a process that updates the movement estimate using a given error, the difference between predicted and sensory results. Although the two updating processes are in different granularity: to update the estimate within one movement trial or between trials, they conceptually produce the same thing. A parameter has been used to weight the error in each of the updating processes: a Kalman gain K in the Kalman filter, and a learning rate η described in Section 3.1 above. Because η has a nature that K does not have—the representation of the effect of memory of errors—it is reasonable to add η into the Kalman filter to weight the error together with K .

Second, the major benefit of adding the parameter η is to handle more cases—allowing us to simulate more human behaviors using memory of errors; the original Kalman filter uses only the previous estimate to make the current estimate. The modified Kalman filter has both inherited the capabilities of the original Kalman filter [1] that simulates the estimation process within a single trial of movement, and obtained a new way to weight the error for updating the estimate of movements [2], so as to simulate the estimation between movement trials. In the following section, we examine the capabilities of the modified Kalman filter by implementing it into a simulated lifting movement.

4 Experiments

In this section, we test the performance of the estimation process of our newly proposed model in a simulated hand lifting action, by comparing its estimation process with human behaviors reported from two previous studies [1, 2]. The comparison results support our new model’s ability to simulate the estimation process not only within one trial of the movement but also between trials using memory of errors.

4.1 Experimental Setup

From recent reviews of the study of human hand-lifting movement [12, 13], we see that some researchers [14-16] have supported the existence of a (forward) internal model occurring during lifting. They hypothesize that people predict their lifting movements based on a system that simulates the behavior of their body and their environment [13], and “the CNS signals the sensory discrepancy between the predicted and actual sensory consequences of action” [14]. These hypotheses have led us to choose lifting as a reasonable target movement to which to apply our model to simulate the human movement estimation process, because the hypotheses support the primary mechanism of our model, a modified Kalman filter.

We use a software robot simulation (youBot), a robot controller (the LIDA[‡] Framework [17]), and a virtual experimental environment (Webots [18]) to simulate a lifting movement. We consider this

[‡] For historical reasons LIDA stands for Learning Intelligent Distribution Agent.

robotic simulation to be a LIDA-based software agent. LIDA is a systems-level cognitive model [19], and the LIDA Framework is a computational implementation of LIDA. We implement our new model into the Sensory Motor System (SMS) of LIDA, which is a module of LIDA simulating the process of action execution[20]. After we have embedded the new model into the controller of the agent, the agent is able to estimate the results of its own actions. Because of space limitations, we leave more details to the Supplementary Materials[§]. Here we present only a screenshot of the LIDA-based agent lifting an object (Fig. 1), so as to give an intuitive feel for the agent and its action. Specifically in our experiment, lifting refers to an action in which the agent grips an object, and moves it upwards. The gripper tip locations serve as the hand locations.

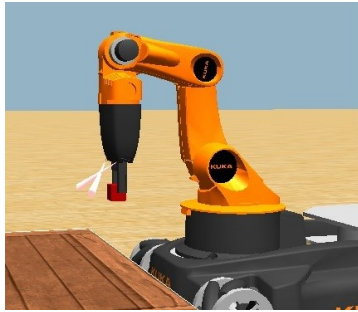


Figure 1: A screen shot of a LIDA-based agent lifting an object

4.2 Implementation of the Learning Rate η

As defined in Section 3.1, the value of η depends on both the number of historical errors and the switching time between these errors. Computationally, we created three variables stored in long-term memory: (1) the number of errors n , (2) the number of switches s , and (3) the current error type c . The first two variables n and s have been introduced in Section 3.1. Variable c is used to determine whether the current error and the upcoming error have different types. If the two errors have different types, one instance of error-switching will be accumulated to variable s ; otherwise the value of s does not change.

In the experiment, these three variables are retrieved once when the agent initializes a lifting movement; thus, the value of η is calculated before the start of the movement and is constant within one trial. Then, at the end of every lifting trial, the three variables are updated based on the error between the estimated hand location and the actual hand location. In this way, the value of η may change between trials.

4.3 Estimation without Memory of Errors

We prepare a computational experiment that is configured similarly to the human experiment reported earlier [1], which studies the estimation process within a single movement trial without being concerned about memory of errors.

As we have reviewed in Section 2.1, in the study of Wolpert and colleagues [1], human participants are asked to move their hand in dark, and they stop moving and report an estimated hand location when they hear a tone. In our simulation, the agent does not have visual sensors but senses the angles of its arm's joints; this configuration conforms to the situation in the human experiment, namely, that participants are without vision, and guided solely by proprioception. Also, we created a program that sends a stop command to the agent, instructing it to stop lifting (see Section 3 in the Supplementary Materials for details); this program plays the role of the experimenters who control the

[§] Find the document here: http://cerg.cs.memphis.edu/assets/papers/2015/MKF_Supplement.pdf

timing of the tone in the human experiment. In the human case, a pair of real and estimated hand locations was collected at the end of each movement trial. So in total, 2400 data pairs were collected (eight participants with 300 trials each). In our simulation, the “experimenter” program generates one stop command at a different virtual time** during each lifting trial. Stop commands are generated so as to give a range of lift durations from 6 to 65 units over 60 lifting trials. We consider the process during the first 5 time units to be the system’s initiation process, and did not collect data during this interval. We performed 40 repetitions of the above trial block (60 lifting trials with different durations) for a total of 2400 data pairs of estimated hand location and actual location, in order to achieve parity with the data collected during the human experiment.

In our simulation, the differences (errors) between real and estimated hand locations are recorded as a function of duration of the hand lifting movement. The average error for each moment (virtual time unit) is calculated, and is shown in Figure 2; movement duration is represented as a number of virtual time units. These simulated results are qualitatively similar to the human data [1]: Overall, 1) the hand location is overestimated, and 2) the error peaks in the first part of the movement (at virtual time 23), and then goes down.

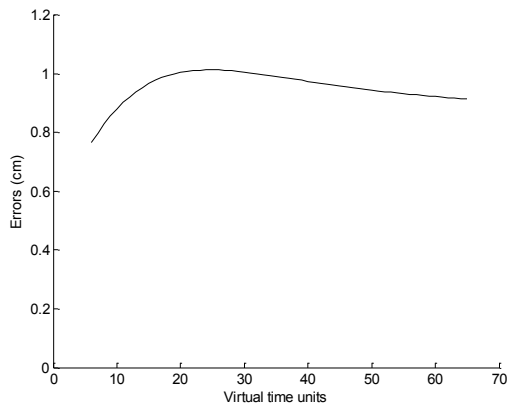


Figure 2: Simulated estimation errors of hand lifting action on average without memory of errors

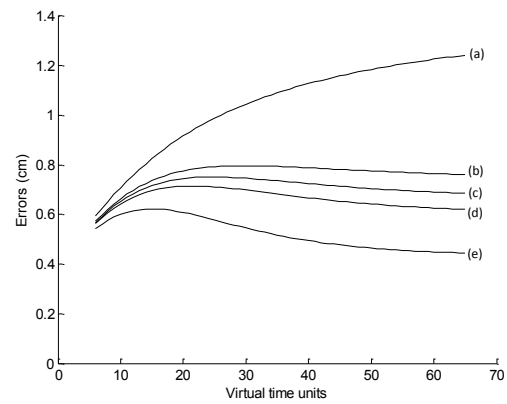


Figure 3: Different propagation of simulated estimation errors of hand lifting action on average. The propagation of errors (a) through (e) are when experiencing different environments that have the error switching rates of 90%, 70%, 50%, 30%, and 10% respectively

4.4 Estimation with Different Memory of Errors

In this sub-section, we describe a computational experiment to examine the effect of memory of errors on the estimation process of an agent. This effect has been examined in, and supported by, human experiments [2] (See Section 2.2).

In our experiment, the agent may lift three types of objects, which have different weights: 0.1kg, 0.2kg, or 0.3kg. We consider 0.2kg to be the default weight, and 0.1kg to be lighter and 0.3kg to be heavier. To artificially create errors as those that were introduced in the human experiments [2], we first configure the agent’s knowledge of the object’s weight to a default value (0.2kg), and then let the agent lift either a lighter (0.1kg) or a heavier object (0.3kg). In this way, the difference (error) occurs between the estimated hand location and the actual one, and two types of errors, positive or negative, are made by using lighter or heavier objects respectively.

** The agent executes at unit intervals in Webots virtual time.

Based on the fact that the sequence of errors stored in memory may switch between positive and negative, we prepare five types of environment that the agent can experience: error switching rates of 10%, 30%, 50%, 70%, or 90% respectively.

To observe the effect of memory of errors, we first let the agent perform 30 lifting trials, using either lighter or heavier objects, to create its memory of errors, and then we let it do one lifting trial using a heavier object. We analyze the propagation of simulated estimation errors during the last lifting trial when the agent has experienced a certain type of environment. In detail, we let the agent perform the above 31 trials 25 times for each type of environment, and calculated the estimation errors on average during the 31st trial, as shown in Figure 3. Within every 31 trials, the value of η changes (see Sections 3.1 and 4.2), and its value is initialized to zero when the agent starts a new sequence of 31 trials. The approach we are using here to explore the effect of memory of errors is based on the design of previous human experiments [2].

As shown in Figure 3, the simulated estimation errors are different in different environments. In detail, the errors are smaller when the environment the agent has experienced has a lower switching rate of errors—propagation (a) is largest and (e) is smallest; that is, the error propagation peak is lower, and the decline after the peak is more rapid. This difference demonstrates that when the environment is more stable—having lower error switching rate—the estimated hand location is closer to the actual because the agent learns more from a given error created using a heavier object. This effect of the environment (memory of errors) matches the phenomenon found in the human experiment [2]. In more detail, the value of η is different while generating propagations of simulated estimation errors ((a) ~ (e)). For example, in situation (a), the agent experiences a rapidly changing environment (switching rate of 90%), so in Eq. (7) variable s is close to n , and then together with Eq. (6) the value of η nearly reaches its minimum, 0.5. On the other hand, in situation (e), because the agent experiences a very steady environment (switching rate of 10%), we can infer that the value of η nearly reaches its maximum, 1.5. Similar computational inferences can be done for situations (b) ~ (d) as well. These inferences match the interpretation of Herzfeld and his colleagues for the human results [2]. Therefore, we argue that we have simulated both the phenomenon and causal factors present in certain human experiments [2].

Furthermore, for most propagations of simulated estimation errors shown in Figure 3, from (b) through (e), their behaviors are similar to study results of human behavior [1]: 1) the hand location is overestimated, and 2) the error peaks in the first part of the movement, and then goes down. The only exception is the propagation (a) in Figure 3, which does not exactly follow the human experimental result [1]: although it shows the overestimation of the hand location, its error simply goes up but does not have an ensuing decline. We think this exception may be due to the fact that the 90% switching rate is an extreme situation that is outside the scope of the hypothesis [1] describing usual human behavior. In this situation, the agent has experienced a very rapidly changing environment, so it almost does not believe the current sensed data—the agent’s knowledge dominates the estimation. That is why the decline does not appear after the peak, and the decline is the result of a trade-off between the agent’s knowledge of the dynamics of the environment and its sensory data.

In summary, together with the experimental results shown in Section 4.3, we have shown that an agent embedded with our newly proposed model is able to simulate both (1) human estimation of its lifting movement within one trial [1], and (2) human estimation between lifting trials driven by memory of errors [2].

5 Future Work

We have presented a new model that estimates human movements using memory of errors. Furthermore, we have computationally embedded this model into a cognitive model, LIDA [19], to

simulate human self-estimation of their movements. In the future, we plan to investigate the impact of this estimation process on the motor control and other cognitive processes within the LIDA model.

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